

Time-periodic solutions of massive scalar fields in AdS background: perturbative constructions

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Abstract

We consider scalar fields which are coupled to Einstein gravity with a negative cosmological constant, and construct periodic solutions perturbatively. In particular, we study tachyonic scalar fields whose mass is at or above the Breitenlohner-Freedman bound in four, five, and seven spacetime dimensions. The critical amplitude of the leading order perturbation, for which the perturbative expansion breaks down, increases as we consider less massive fields. We present various examples including a model with a self-interacting scalar field which is derived from a consistent truncation of IIB supergravity.

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I. INTRODUCTION

In the AdS/CFT correspondence [1], one usually relates a strongly interacting quantum field theory with a classical anti-de Sitter Einstein gravity with a negative cosmological constant with matter fields. Replacing a quantum field theory with a classical equation of motion is certainly a great simplification, but the price to pay is that one has to go to a higher dimensional spacetime. In broad terms, the dependence on the *radial* direction in the gravity provides the scale dependence of physical quantities. A particularly nice property of the AdS/CFT is that black holes are dual to field theory at finite temperature, so time-dependent process on the gravity side can in principle describe time evolution of a thermal system. The quantitative understanding of black hole formation within AdS space is thus certainly desirable.

Recently several groups have studied numerically the formation of a black hole in AdS space with a matter field. A seminal paper along this direction is [2] (see also [3, 4]), where the authors presented numerical solutions of the coupled nonlinear partial differential equations from Einstein-massless-scalar field system with a spherically symmetric ansatz. The conclusion drawn from the data is that AdS spacetime is generically unstable under small perturbations of matter fields, due to nonlinearity which transfers energy to higher frequency modes. However, it was discovered soon that there exist many nonlinearly stable solutions [5] and also time-periodic solutions in AdS space [6]. The authors of [6] considered a massless scalar field in AdS₅ space and solved the field equation perturbatively and argued for the existence of periodic solutions. Cancellation of secular terms through a shift of the frequency is an essential part of the construction. For related works readers are referred to [7–23].

The aim of this work is to extend the study of time-dependent solutions in gravity-scalar system to tachyonic fields. In most of the previous works, probably for definiteness and simplicity, the authors chose massless scalar fields. As it is well known however, in AdS space “massless” field is not exactly at the border of stability, which is usually called the Breitenlohner-Freedman (BF) bound. Stability requirement of a scalar field in AdS_{*d*+1} for instance is in fact $m^2 \geq -\frac{d^2}{4\ell^2}$, where ℓ is the curvature radius. According to the AdS/CFT correspondence, tachyonic scalars above the BF bound are dual to relevant operators, while a massless scalar field is dual to a marginal operator. It is thus an obviously impending

question: whether a tachyonic scalar can also lead to periodic solutions, and if the answer is yes how much quantitative and qualitative difference they have, compared to massless scalars. In the next Section we report the result of our symbolic computation. For all the tachyonic scalar fields we have considered we have checked the cancellation of secular terms and explicitly obtained periodic solutions perturbatively. As it is naturally expected, the radius of convergence for the amplitude of perturbation field becomes larger as we consider large values of $(-m^2)$ values.

II. THE GRAVITY-SCALAR SYSTEM AND ITS PERTURBATIVE SOLUTIONS

Our starting point is the following action of a massive real scalar field field coupled to Einstein gravity with a cosmological constant Λ . (We note that we closely follow the convention of [2].)

$$S = \int d^{d+1}x \sqrt{-g} \left(\frac{1}{16\pi G} (R - 2\Lambda) - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 \right). \quad (1)$$

The spacetime is $d+1$ dimensional, and we consider $\Lambda < 0$, i.e. the vacuum is anti-de Sitter. We take a spherically symmetric ansatz, and more concretely the metric is written as

$$ds^2 = \frac{\ell^2}{\cos^2 x} \left(-Ae^{-2\delta} dt^2 + \frac{dx^2}{A} + \sin^2 x d\Omega_{d-1} \right). \quad (2)$$

Here the metric component fields A, δ , as well as the matter field ϕ , depend only on t, x . $d\Omega_{d-1}$ denotes the line element of the $(d-1)$ -dimensional unit sphere. The curvature radius ℓ is determined as $\Lambda = -\frac{d(d-1)}{2\ell^2}$.

When one computes the Einstein tensor from the metric ansatz above, at first sight it looks like there are four non-vanishing and independent components, e.g. G_{tt}, G_{tr}, G_{rr} and the components on the sphere S^{d-1} . But two of them are in fact constraints, which are shown to follow from the remaining equations. This is of course related to the fact that we have allowed non-trivial dependences on two coordinates t, r and their diffeomorphism freedom.

The scalar equation of motion is given as

$$\partial_t(e^\delta A^{-1} \partial_t \phi) - \frac{1}{\tan^{d-1} x} \partial_x (Ae^{-\delta} \tan^{d-1} x \partial_x \phi) + \frac{\Delta(\Delta - d)}{\cos^2 x} e^{-\delta} \phi = 0. \quad (3)$$

Here we set $m^2 = \Delta(\Delta - d)/\ell^2$ and assume that the mass parameter is above the Breitenlohner-Freedman bound, i.e. $m^2 \geq -\frac{d^2}{4\ell^2}$. Here $\Delta \geq d/2$ is the conformal dimension of the dual operator through AdS/CFT correspondence.

The two independent equations from the variation of metric are

$$\delta' = -\sin x \cos x (A^{-2} e^{2\delta} \dot{\phi}^2 + \phi'^2), \quad (4)$$

$$A' = A\delta' + \frac{d-2+2\sin^2 x}{\sin x \cos x} (1-A) - \frac{\Delta(\Delta-d)\sin x}{\cos x} \phi^2. \quad (5)$$

We can solve the equations perturbatively around the vacuum AdS solution $A=1, \delta=0$ and $\phi=0$. At first order, we set $\phi = \varepsilon \phi^{(1)}$ for a small parameter ε . If we use the usual technique of separation of variables $\phi^{(1)} = f(x) \cos \omega t$ the scalar equation (3) gives a Sturm-Liouville problem $Lf(x) = \omega^2 f(x)$ with

$$Lf(x) \equiv -\frac{1}{\tan^{d-1} x} \frac{d}{dx} \left[\tan^{d-1} x \frac{df}{dx} \right] + \frac{\Delta(\Delta-d)}{\cos^2 x} f(x). \quad (6)$$

It is straightforward to solve this equation. The eigenfunctions and the eigenvalues are

$$e_j(x) = 2 \sqrt{\frac{(j+\Delta/2)\Gamma(j+1)\Gamma(j+\Delta)}{\Gamma(j+d/2)\Gamma(j+\Delta-d/2+1)}} (\cos x)^\Delta P_j^{d/2-1, \Delta-d/2}(\cos 2x), \quad (7)$$

$$\omega_j = 2j + \Delta. \quad (8)$$

Here $P_j^{a,b}(u)$, $j=0,1,2,\dots$ are Jacobi polynomials. We note that the eigenfunctions are normalized as

$$\int_0^{\pi/2} e_i(x) e_j(x) \tan^{d-1} x dx = \delta_{ij}. \quad (9)$$

At the next order $\mathcal{O}(\varepsilon^2)$, we can easily solve (4),(5) and obtain $A = 1 - \varepsilon^2 A^{(2)}, \delta = \varepsilon^2 \delta^{(2)}$. We choose the convention $\delta(t, x=0) = 1 - A(t, x=0) = 0$ for integration constants. More concretely, when we integrate (4)

$$\delta^{(2)}(t, x) = - \int_0^x \sin y \cos y \left((\partial_t \phi^{(1)}(t, y))^2 + (\partial_y \phi^{(1)}(t, y))^2 \right) dy. \quad (10)$$

Similarly we get

$$\begin{aligned} A^{(2)}(t, x) = & \frac{\cos^d x}{\sin^{d-2} x} \int_0^x \tan^{d-1} y \left[(\partial_t \phi^{(1)}(t, y))^2 + (\partial_y \phi^{(1)}(t, y))^2 \right. \\ & \left. + \frac{\Delta(d-\Delta)}{\sin y \cos y} (\phi^{(1)}(t, y))^2 \right] dy. \end{aligned} \quad (11)$$

In the next order $\mathcal{O}(\varepsilon^3)$ we need to solve the scalar equation which now becomes an in-homogeneous second order differential equation.

$$\begin{aligned} (\partial_t^2 + L)\phi^{(3)}(t, x) = & \frac{\Delta(\Delta-d)}{\cos^2 x} \delta^{(2)} \phi^{(1)} - \partial_t \left[(\delta^{(2)} + A^{(2)}) \partial_t \phi^{(1)} \right] \\ & - \frac{1}{\tan^{d-1} x} \partial_x \left[(\delta^{(2)} + A^{(2)}) \partial_x \phi^{(1)} \right]. \end{aligned} \quad (12)$$

Here the point is that on the right hand side of the above equation there appears a product of three harmonic functions like $(\cos \omega t)^3$. Using the elementary algebra of trigonometric functions, it gives rise to *secular* modes whose frequency is the same as one of the original frequencies $\omega_j = \Delta + 2j$. Naively this means that the amplitude of the resonant modes increases linearly with time, but as it is well known this kind of instability is unphysical if it can be absorbed by shifting the frequency $\omega \rightarrow \omega + \varepsilon^2 \omega^{(2)}$. It has been verified in [6] that, for $d = 4$ (AdS₅) and a massless scalar field, if we start with a single mode at $\mathcal{O}(\varepsilon)$ the secular terms are cancelled perturbatively up to fairly high orders in ε . For AdS₅ and the the lowest lying mode $j = 0$, the frequency as a function of the perturbative parameter ε is found as

$$\Omega = 4 + \frac{464}{7}\varepsilon^2 + \frac{45614896}{11319}\varepsilon^4 + \dots \quad (13)$$

In [6] it is reported that the coefficients were obtained up to ε^{16} . Through the Padé approximation the series seems to be convergent with radius of convergence $\varepsilon \approx 0.09$.

From the analytic expression of the perturbative solution, we may extract a lot of data which can help us understand the time-evolution of our solution. Let us take the function A for example. It is obvious that $A = 0$ at a particular point in the spacetime implies the formation of a black hole. From the expression for A which is exact up to the order of $\mathcal{O}(\varepsilon^{20})$ we have created plots for the time-oscillation for different values of ε . The minimum of A decreases for larger ε , and if we extrapolate our perturbative solution to bigger values of ε , A hits zero at $\varepsilon \approx 0.11$.

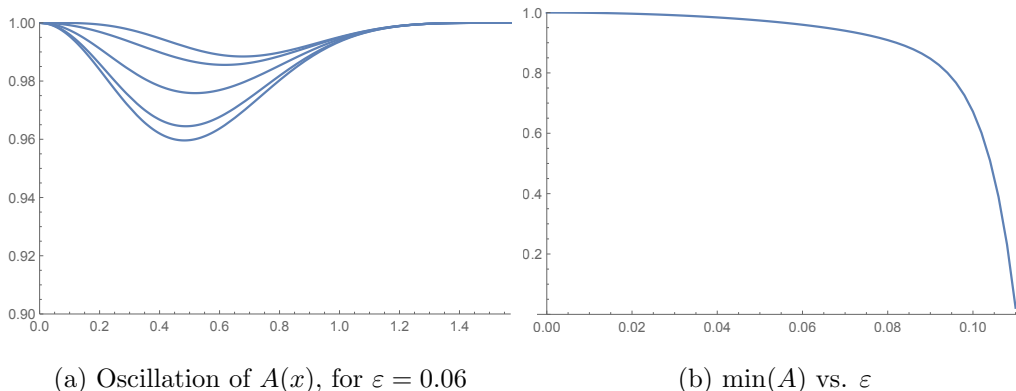


FIG. 1: The plot in the left panel shows the oscillation of $A(t, x)$. The minimum value of A decreases for larger ε . The plot in the right panel shows $\min(A)$ as a function of ε .

A. Massive scalars in AdS₅

We have written a code which constructs periodic solutions perturbatively in *Mathematica* and have confirmed the result for the case of a massless scalar field in AdS₅ agrees with [6]. In fact we pushed the computation to $\mathcal{O}(\varepsilon^{20})$: the coefficients of $\varepsilon^{18}, \varepsilon^{20}$ in (13) are approximately $3.92591 \times 10^{17}, 4.45447 \times 10^{19}$. The Padé approximation at (10, 10) then gives the pole of the denominator at $\varepsilon = 0.0904562$. On our laptop with an 2.7GHz Intel i7 CPU and 16 GB RAM, the last step of calculating this coefficient at $\mathcal{O}(\varepsilon^{20})$ took less than 3 hours and 10 minutes.

The first choice of our own is a scalar field exactly at the BF bound ($\Delta = 2$, or equivalently $m^2 = -4/\ell^2$). For the lowest lying mode the probe limit gives eigenfrequency $\omega = 2$. Our perturbative algorithm gives

$$\begin{aligned} \Omega = & 2 + \frac{62}{15}\varepsilon^2 + \frac{31373}{2250}\varepsilon^4 + \frac{1757780088437}{24169635000}\varepsilon^6 + \frac{537359617120101825761}{1278264665452500000}\varepsilon^8 \\ & + \frac{1268572361264125960914631343583143413}{483709750330891209012418781250000}\varepsilon^{10} \\ & + \frac{1459283228526801137059175769554860613450261076853469483}{85131519913245630307027620371897681405325000000000}\varepsilon^{12} + \dots \quad (14) \end{aligned}$$

We have obtained the coefficients of up to ε^{20} . They all turn out to be rational numbers whose numerator and denominator have too many digits to be explicitly reported here. The coefficients of $\varepsilon^{14}, \varepsilon^{16}, \varepsilon^{18}, \varepsilon^{20}$ are approximately $1.1582059 \times 10^5, 8.0214964 \times 10^5, 5.6628666 \times 10^6, 4.0593615 \times 10^7$.

Based on this result we also performed the Padé approximation. From (4, 4) up to (10, 10), the (smallest) zero of the denominator is respectively 0.411687, 0.380910, 0.368009, 0.360290.

We have repeated a similar computation for $\Delta = 3$ or $m^2 = -3/\ell^2$.

$$\begin{aligned} \Omega = & 3 + \frac{297}{14}\varepsilon^2 + \frac{11388681}{27440}\varepsilon^4 + \frac{75814410351189977829}{6895049537868800}\varepsilon^6 \\ & + \frac{440953730050912073171536929147}{1332903538719975878656000}\varepsilon^8 \\ & + \frac{108962184535866721154985183970785991244847410023180150827}{10174886986824762523197846218808661282081341440000}\varepsilon^{10} + \dots \quad (15) \end{aligned}$$

For the next coefficients our result gives for the coefficients of $\varepsilon^{12}, \varepsilon^{14}, \varepsilon^{16}, \varepsilon^{18}, \varepsilon^{20}$ approximate values $3.6360632 \times 10^8, 1.2769397 \times 10^{10}, 4.5987515 \times 10^{11}, 1.6888399 \times 10^{13}, 6.2996028 \times 10^{14}$. The Padé approximation gives that the upper bound for the perturbative approach to be well-behaved is $\varepsilon \approx 0.157957$.

An interesting variation of this system is given by a truncation of supergravity, from the solutions of the form $AdS_5 \times X^5$ in IIB supergravity where X^5 is a Sasaki-Einstein manifold [24]. In particular, a further truncated theory with a vector field and two real scalars with a nontrivial potential function has been used to address holographic superconductors [25]. For our purpose we turn off the vector field as well as the axion. Then the scalar potential adjusted to our convention is written as

$$V(\phi) = -\frac{1}{2\ell^2} \left(-4 + \cosh^2 \frac{\sqrt{6}\phi}{2} \left(5 - \cosh \sqrt{6}\phi \right) \right) \quad (16)$$

In the small field limit the mass of the scalar corresponds to $\Delta = 3$. We have confirmed that the cancellation of secular terms persist also in this supergravity-inspired model.

$$\begin{aligned} \Omega = & 3 + \frac{837}{35}\varepsilon^2 + \frac{18123993}{42875}\varepsilon^4 + \frac{1022167072159904258901}{102073404519520000}\varepsilon^6 \\ & + \frac{1885826584327612453573347913521573}{6924534314390082309920000000}\varepsilon^8 \\ & + \frac{4854622063875589224275650949019735931691812729968299301296434317}{606704146497857938746558919904014257651677884288000000000}\varepsilon^{10} \\ & + \dots \end{aligned} \quad (17)$$

We have also obtained more coefficients up to ε^{20} : they are 2.4695488×10^8 , 7.8888024×10^9 , 2.5852724×10^{11} , 8.6415084×10^{12} , 2.9344207×10^{14} . The Padé approximation at $(10, 10)$ gives that the radius of convergence for ε is 0.165696 and there is no huge difference from the previous example of $\Delta = 3$.

B. Massless and massive scalars in AdS_7

We can repeat the same analysis for $d = 6$ case. Again at any order of the perturbative computation the fields are expressed as a finite order polynomial of $u = \cos x$. For a massless scalar field, in the probe limit the eigen-frequency is $\omega = 6$. Explicit computation gives

$$\begin{aligned} \Omega = & 6 + \frac{133920}{143}\varepsilon^2 + \frac{204857013644928}{347980633}\varepsilon^4 \\ & + \frac{6653917224931500928527205438989898}{13966872826194750738453177}\varepsilon^6 \\ & + \frac{1080461333185973832680808465193246135959458551807301}{2505052939963175783424913698557925050655}\varepsilon^8 + \dots \end{aligned} \quad (18)$$

We obtained the coefficients up to ε^{20} . The coefficients of $\varepsilon^{10}, \dots, \varepsilon^{20}$ are 4.16417×10^{14} , 4.1925×10^{17} , 4.34786×10^{20} , 4.60964×10^{23} , 4.97161×10^{26} , 5.43594×10^{29} . The

Padé approximation at (n, n) for $n = 2, \dots, 10$ gives the poles of the denominator at 0.032326, 0.0304412, 0.029661, 0.0292566. When compared to the case of AdS_5 , the coefficients are larger and the pole of Padé approximant is smaller. This means that the perturbative expansion breaks down more easily for small amplitude of $\phi^{(1)}$. Another way to see this is to check how many modes are turned on for a specific order of ε . At ε^{20} , the scalar field includes e_{70} . On the other hand, for AdS_5 the highest mode at ε^{20} is e_{50} .

We have repeated the computation for a tachyonic scalar field with $\Delta = 3, 4, 5$. Firstly for $\Delta = 3$, or $m^2 = -9/\ell^2$.

$$\begin{aligned} \Omega = & 3 + \frac{3807}{280}\varepsilon^2 + \frac{2704629609}{21952000}\varepsilon^4 + \frac{22814710893326488039461}{14774928824320000000}\varepsilon^6 \\ & + \frac{11684631773098620212295629421580959}{544768173672961638400000000000}\varepsilon^8 + \dots \end{aligned} \quad (19)$$

And the next coefficients for $\varepsilon^{10}, \dots, \varepsilon^{20}$ are $3.16957 \times 10^5, 4.87182 \times 10^6, 7.69766 \times 10^7, 1.24135 \times 10^9, 2.03364 \times 10^{10}, 3.37368 \times 10^{11}$. The Padé approximation at (n, n) for $n = 4, 6, 8, 10$ exhibit a pole at 0.264111, 0.248389, 0.242002, 0.238681.

Secondly for $\Delta = 4$, or $m^2 = -8/\ell^2$. The frequency is given as

$$\begin{aligned} \Omega = & 4 + \frac{3152}{35}\varepsilon^2 + \frac{24139995472}{4244625}\varepsilon^4 + \frac{89200146157625691820278256}{190178211481119736875}\varepsilon^6 \\ & + \frac{961459118126637937051446867780955648086736}{22263941138510438405094532804453125}\varepsilon^8 + \dots \end{aligned} \quad (20)$$

The next coefficients for $\varepsilon^{10}, \dots, \varepsilon^{20}$ are $4.23236 \times 10^9, 4.31887 \times 10^{11}, 4.53397 \times 10^{13}, 4.86117 \times 10^{15}, 5.29766 \times 10^{17}, 5.84892 \times 10^{19}$. The Padé approximation at (n, n) for $n = 4, 6, 8, 10$ exhibit a pole at 0.101701, 0.0958928, 0.0935393, 0.0923499.

Finally for $\Delta = 5$, or $m^2 = -5/\ell^2$. The frequency is given as

$$\begin{aligned} \Omega = & 5 + \frac{103375}{308}\varepsilon^2 + \frac{1065400702671875}{13674076416}\varepsilon^4 \\ & + \frac{867158669318199310085443515535234375}{37159737704925947186764136448}\varepsilon^6 \\ & + \frac{2031755203183353658899088230995968728695683830185546875}{260115931529596661308343233550649096051621888}\varepsilon^8 + \dots \end{aligned} \quad (21)$$

And the next coefficients for $\varepsilon^{10}, \dots, \varepsilon^{20}$ are $2.78552 \times 10^{12}, 1.03514 \times 10^{15}, 3.96003 \times 10^{17}, 1.54806 \times 10^{20}, 6.15393 \times 10^{22}, 2.4793 \times 10^{25}$. And the Padé approximation at (n, n) for $n = 4, 6, 8, 10$ exhibit a pole at 0.0532139, 0.0501531, 0.0488935, 0.0482435.

C. Massless scalar in AdS₄

In this subsection we address the case of an odd d , in particular AdS₄. The general analysis here is rather cumbersome, because from the next order in perturbation at $\mathcal{O}(\varepsilon^2)$ the perturbative fields are not given as a polynomial in $u = \cos x$. This means that we have to deal with a summation over all the eigenmodes. However, one can show that at $\mathcal{O}(\varepsilon^3)$ the secular modes can be removed through a shift of the frequency in the scalar equation, just like previous examples.

More concretely let us consider perturbation with a massless scalar field which has the smallest frequency, $\omega = 3$.

$$\phi^{(1)} = \varepsilon e_0(u) = \varepsilon \sqrt{\frac{32}{\pi}} u^3 \cos(3t). \quad (22)$$

Then from (10) we obtain

$$\delta^{(2)} = \frac{12}{\pi} (2(u^6 - 1) - (3u^8 - 2u^6 - 1) \cos(6t)) , \quad (23)$$

which can be expressed as a linear combination of eigenmodes $e_j(u)$. For the function A however, we obtain as a function of $u = \cos x$ given as follows:

$$A^{(2)} = \frac{6u^3}{\pi} \left(\frac{3 \cos^{-1} u}{\sqrt{1-u^2}} + u (3 - 6u^2 + 8u^2(u^2 - 1) \cos(6t)) \right) . \quad (24)$$

This obviously involves an infinite sum over the eigenmodes $e_j(u)$ in (7), which are all polynomials in u . The expansion coefficients for the right hand side of (12) as $\sum_{j=0}^{\infty} f_j(t) e_j(u)$ can be worked out and the result is

$$f_0(t) = -\frac{459(8 \cos 3t - 5 \cos 9t)}{16\pi} , \quad (25)$$

$$f_1(t) = \frac{9\sqrt{3}(1374 \cos 3t - 595 \cos 9t)}{160\pi} , \quad (26)$$

$$f_2(t) = \frac{9\sqrt{6}(59 \cos 3t - 140 \cos 9t)}{160\pi} , \quad (27)$$

$$f_3(t) = \frac{14607\sqrt{10} \cos 3t}{5600\pi} , \quad (28)$$

$$f_4(t) = \frac{9\sqrt{15}(202 \cos 3t + 175 \cos 9t)}{5600\pi} , \quad (29)$$

$$f_j(t) = \frac{162\sqrt{2}}{\pi} \cdot \frac{(-1)^j (2j+3)(j^2+3j+8) \cos 3t}{j(j-1)(j+3)(j+4)(j+1)^{3/2}(j+2)^{3/2}}, \quad j \geq 5. \quad (30)$$

The appearance of $\cos 3t$ and $\cos 9t$ is easy to understand, since at $\mathcal{O}(\varepsilon^3)$ we are dealing with $\cos^3 3t$. If we recall that the eigenfrequency for e_j is $\omega_j = 3 + 2j$, potentially there can be resonances for $\omega_0 = 3$ and $\omega_3 = 9$. But as we see in the above, f_3 does *not* contain $\cos 9t$: this rather miraculous cancellation of secular terms applies to all other examples discussed in [6][18] and this paper so far. f_0 contains a resonance term, but we can cancel it through renormalization of the frequency $\omega \rightarrow \Omega = \omega + \varepsilon^2 \omega^{(2)}$, with

$$\Omega = 3 + \frac{153}{4\pi} \varepsilon^2. \quad (31)$$

Integration of (12) is now straightforward using the technique of separation of variables. It will be interesting to compute higher order terms in Ω , but we will leave it for a future work.

III. DISCUSSION

We have so far analysed the perturbative computation of classical scalar-Einstein gravity equations by extending previous works on massless scalars to the case of tachyonic ones. This work stands also as a technical improvement, since we have pushed the perturbative expansion to $\mathcal{O}(\varepsilon^{20})$, while Ref.[6] reported results upto $\mathcal{O}(\varepsilon^{16})$. Our result confirms that the periodic solutions and the associated removal of secular terms in [6] persist for massive scalars. The central quantitative result of ours is the change of frequency renormalization as a function of the mass of the scalar fields. We confirmed the natural prediction that the perturbative series should be valid for larger amplitudes as we decrease m^2 .

It will be interesting if one can generalize our analysis to non-integer values of Δ , but in that case - just like a massless scalar in AdS_4 - higher order configurations in general cannot be expressed as a finite sum over the normal modes of the probe scalar equations so it will be difficult to automatize the computation. It will be very nice if we can find the exact mass dependence of the radius of convergence for the perturbation parameter ε , for general Δ and d . It will be also interesting to study different matter fields or modified gravity theories, for instance Gauss-Bonnet theory which through AdS/CFT correspondence corresponds to $1/N$ corrections on the dual field theory side.

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- [1] J. M. Maldacena, *The Large N limit of superconformal field theories and supergravity*, *Adv.Theor.Math.Phys.* **2** (1998) 231–252, [[hep-th/9711200](#)].
 - [2] P. Bizon and A. Rostworowski, *On weakly turbulent instability of anti-de Sitter space*, *Phys.Rev.Lett.* **107** (2011) 031102, [[arXiv:1104.3702](#)].
 - [3] O. J. Dias, G. T. Horowitz, and J. E. Santos, *Gravitational Turbulent Instability of Anti-de Sitter Space*, *Class.Quant.Grav.* **29** (2012) 194002, [[arXiv:1109.1825](#)].
 - [4] S. Stotyn, M. Park, P. McGrath, and R. B. Mann, *Black Holes and Boson Stars with One Killing Field in Arbitrary Odd Dimensions*, *Phys.Rev.* **D85** (2012) 044036, [[arXiv:1110.2223](#)].
 - [5] O. J. Dias, G. T. Horowitz, D. Marolf, and J. E. Santos, *On the Nonlinear Stability of Asymptotically Anti-de Sitter Solutions*, *Class.Quant.Grav.* **29** (2012) 235019, [[arXiv:1208.5772](#)].
 - [6] M. Maliborski and A. Rostworowski, *Time-Periodic Solutions in an Einstein AdS –Massless-Scalar-Field System*, *Phys.Rev.Lett.* **111** (2013) 051102, [[arXiv:1303.3186](#)].
 - [7] G. Fodor, P. Forgacs, and M. Mezei, *Boson stars and oscillatons in an inflationary universe*, *Phys.Rev.* **D82** (2010) 044043, [[arXiv:1007.0388](#)].
 - [8] D. Garfinkle and L. A. Pando Zayas, *Rapid Thermalization in Field Theory from Gravitational Collapse*, *Phys.Rev.* **D84** (2011) 066006, [[arXiv:1106.2339](#)].
 - [9] P. Grandclement, G. Fodor, and P. Forgacs, *Numerical simulation of oscillatons: extracting the radiating tail*, *Phys.Rev.* **D84** (2011) 065037, [[arXiv:1107.2791](#)].
 - [10] J. Jalmuzna, A. Rostworowski, and P. Bizon, *A Comment on AdS collapse of a scalar field in higher dimensions*, *Phys.Rev.* **D84** (2011) 085021, [[arXiv:1108.4539](#)].

- [11] H. de Oliveira, L. A. Pando Zayas, and E. Rodrigues, *A Kolmogorov-Zakharov Spectrum in AdS Gravitational Collapse*, *Phys.Rev.Lett.* **111** (2013), no. 5 051101, [[arXiv:1209.2369](#)].
- [12] A. Buchel, S. L. Liebling, and L. Lehner, *Boson stars in AdS spacetime*, *Phys.Rev.* **D87** (2013), no. 12 123006, [[arXiv:1304.4166](#)].
- [13] M. Maliborski and A. Rostworowski, *Lecture Notes on Turbulent Instability of Anti-de Sitter Spacetime*, *Int.J.Mod.Phys.* **A28** (2013) 1340020, [[arXiv:1308.1235](#)].
- [14] G. Fodor, P. Forgcs, and P. Grandclment, *Scalar field breathers on anti-de Sitter background*, *Phys.Rev.* **D89** (2014) 065027, [[arXiv:1312.7562](#)].
- [15] J. Abajo-Arrastia, E. da Silva, E. Lopez, J. Mas, and A. Serantes, *Holographic Relaxation of Finite Size Isolated Quantum Systems*, *JHEP* **1405** (2014) 126, [[arXiv:1403.2632](#)].
- [16] M. Maliborski and A. Rostworowski, *What drives AdS unstable?*, *Phys.Rev.* **D89** (2014) 124006, [[arXiv:1403.5434](#)].
- [17] V. Balasubramanian, A. Buchel, S. R. Green, L. Lehner, and S. L. Liebling, *Holographic Thermalization, stability of AdS, and the Fermi-Pasta-Ulam-Tsingou paradox*, *Phys.Rev.Lett.* **113** (2014) 071601, [[arXiv:1403.6471](#)].
- [18] B. Craps, O. Evnin, and J. Vanhoof, *Renormalization group, secular term resummation and AdS (in)stability*, *JHEP* **1410** (2014) 48, [[arXiv:1407.6273](#)].
- [19] P. Basu, C. Krishnan, and A. Saurabh, *A Stochasticity Threshold in Holography and the Instability of AdS*, [arXiv:1408.0624](#).
- [20] G. T. Horowitz and J. E. Santos, *Geons and the Instability of Anti-de Sitter Spacetime*, [arXiv:1408.5906](#).
- [21] N. Deppe, A. Kolly, A. Frey, and G. Kunstatter, *Stability of AdS in Einstein Gauss Bonnet Gravity*, [arXiv:1410.1869](#).
- [22] F. V. Dimitrakopoulos, B. Freivogel, M. Lippert, and I.-S. Yang, *Instability corners in AdS space*, [arXiv:1410.1880](#).
- [23] R. Baier, H. Nishimura, and S. Stricker, *Scalar field collapse with negative cosmological constant*, [arXiv:1410.3495](#).
- [24] J. P. Gauntlett, S. Kim, O. Varela, and D. Waldram, *Consistent supersymmetric Kaluza-Klein truncations with massive modes*, *JHEP* **0904** (2009) 102, [[arXiv:0901.0676](#)].
- [25] S. S. Gubser, C. P. Herzog, S. S. Pufu, and T. Tesileanu, *Superconductors from Superstrings*, *Phys.Rev.Lett.* **103** (2009) 141601, [[arXiv:0907.3510](#)].